of x' and y' differ from x and y respectively by infinitely small quantities. It is easy to see that the coefficients of $\delta\alpha$ do not vanish, for if we put for $\varphi(x, y, \alpha)$ and $\psi(x, y, \alpha)$ their equals x_1 and y_1 respectively, these coefficients equated to zero are

$$\frac{\partial\varphi(x_1, y_1, \bar{a})}{\partial\bar{a}}\equiv 0, \quad \frac{\partial\psi(x_1, y_1, \bar{a})}{\partial\bar{a}}\equiv 0.$$

But these last identities assert that φ and ψ are free from $\bar{a}$, that is, in general the equations of the group contain no parameter which is contrary to hypothesis.

The quantity $\bar{a}$ is a function of α, since to a transformation $(\bar{a})$ there corresponds, by hypothesis, a completely determinate inverse transformation $(\bar{a})$. The equations (1) of the infinitesimal transformation may be written in the form

$$x'=x+\xi(x, y, \alpha)\delta\alpha+\ldots\ldots, \quad y'=y+\eta(x, y, \alpha)\delta\alpha+\ldots\ldots*$$

LIE thus arrives at the following theorem :

I. *Every one parameter group whose transformations are inverse in pairs contains at least one infinitesimal transformation.*

Princeton University, 22 October, 1897.

[To be Continued.]

ARITHMETIC.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

83. Proposed by the late REV. G. W. BATES, A. M., Pastor of M. E. Church, Dresden City, Ohio.

A has three notes; the first and second, \$1000 each, and the third \$457; all dated April 1, 1884. The first is due April 1, 1888, second, April 1, 1889, and the third, April 1, 1890, and each bearing interest at 6%. What must B pay for the three notes September 21, 1886, that the investment will bring him 8% compound interest ?

Solution by G. B. M. ZERR, A. M., Ph. D., President of Russell College, Lebanon, Mo.

(I). Regarding the notes as bearing simple interest. We get

\$1000 × 1.24=\$1240, amount of first note.

\$1000 × 1.30=\$1300, amount of second note.

\$457 × 1.36=\$621.52, amount of third note.

*These equations contain a constant a which can be arbitrarily chosen, hence we can find an infinitesimal transformation of the group in many different ways. But the sequel will show that all these, excepting a constant factor, are identical in their terms of the first order of infinitesimals.

From September 21, 1886, to April 1, 1888, is $1\frac{19}{36}$ years.
From September 21, 1886, to April 1, 1889, is $2\frac{19}{36}$ years.
From September 21, 1886, to April 1, 1890, is $3\frac{19}{36}$ years.
Let x=amount paid for first note ; y, for second ; z, for third.
$\therefore x(1.08)^{1\frac{19}{36}}=1240$, or $\log x=\log 1240-1\frac{19}{36}\log 1.08$.
$\therefore x=\$1102.448$.
$y(1.08)^{2\frac{19}{36}}=1300$, or $\log y=\log 1300-2\frac{19}{36}\log 1.08$.
$\therefore y=\$1070.176$.
$\log z=\log 621.52-3\frac{19}{36}\log 1.08$.
$\therefore z=\$473.743$.
$x+y+z=\$2646.367$=whole amount to be paid for the notes.
(II). If the notes bear compound interest we get,
$\$1000\times(1.06)^4=\$1262.477$, amount of first note.
$\$1000\times(1.06)^5=\$1338.226$, amount of second note.
$\$457\times(1.06)^6=\$648.263$, amount of third note.
$\therefore \log x=\log 1262.477-1\frac{19}{36}\log 1.08$.
$\therefore x=\$1122.43$.
$\log y=\log 1338.226-2\frac{19}{36}\log 1.08$.
$\therefore y=1101.646$.
$\log z=\log 648.263-3\frac{19}{36}\log 1.08$.
$\therefore z=\$494.127$.
$x+y+z=\$2718.20$=whole amount paid for the three notes.

GEOMETRY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

78. **Proposed by J. A. MOORE, Ph. D., Professor of Mathematics, Millsaps College, Jackson, Miss.**

Required the number of normals that can be drawn from any point (a, b) to the parabola $y^2=2px$.

I. **Solution by the PROPOSER.**

The equation of the normal to the parabola in terms of its slope, (s), is

$$y=sx-\tfrac{1}{2}(sp)(2+s^2)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1).$$

Substituting a, b for x, y in (1), and putting the equation in a new form, we have,

$$s^3+\tfrac{1}{2}p(p-a)s+(2b/p)=0\ldots\ldots\ldots\ldots\ldots\ldots(2).$$